Bankim Sardar College

Semester II Examination

B.Sc. Gen

Subject: Mathematics

Paper – CC2

Answers of each group should be in separate answer-sheet

Group – A (F.M. 10)

Answer any five questions 5×2

1. (i) Show that the vectors $(1,1-3),(1,2,1)$ and $(7,-4,1)$ are mutually perpendicular.

(ii) Define twin primes. Give an example.

(iii) Find the condition that the equation $ax + by = c$ has an integral solution, a,b,c are integers and a,b are not both zero.

(iv) Find g.c.d. of 858 and 325 and express it in the form $858m + 325n$ $(m, n$ integers).

(v) If $f(x) = tanx$, then examine whether Rolle's theorem is applicable on $(0,\pi)$ or not.

(vi) Find the maximum and minimum value (if any) of the function $f(x) = 4 - |cosx|$.

(vii) Form a partial differential equation by eliminating arbitrary constants a and b from the relation: $z = (x+a)(y+b)$.

$(viii)$ Give an example of Cauchy-Euler differential equation.

Group – B (F.M. 15)

2. (i) Define Perfect Number. Give an example of it. 3

Or

If $f'(c)$ does not exist, can $f(x)$ have a local extremum at $x = c$? Justify your answer.

(ii) If k is a positive integer find the $gcd(ka, kb)$. 2

Or

Write down the necessary conditions for maxima and minima for functions of two variables.

(iii) Find the least positive integer for which $99 \times 101 \times 125$ is congruent to module 11. 3

Or

Evaluate: $\lim\limits_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - sinx}$.

(iv) Find the value of p for which the vectors $-\vec{i} + 5\vec{j} + \vec{k}$ and $p\vec{i} + 2\vec{j} + 3\vec{k}$ perpendicular to each other. 2

Or

Find the complementary function of $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = cos7x$.

(v) Find the magnitude of $\vec{a}+\vec{b}$ where $\vec{a} = \vec{\imath}+2\vec{\jmath}-\vec{k}$ and $\vec{b} = \vec{\jmath}+\vec{k}$. 2

Or

Find the Wronskian of $y_1(x) = x^2$ and $y_2(x) = x^3$.

(vi) Prove that $n(n+1)(n+2)$ is divisible by 6, when n is any integer. 2

Or

Find the value of θ from the Mean Value Theorem $f(a+h) = f(a) + hf'(a+\theta h), \theta \in (0,1)$, where $f(x) = x^2$.

(vii) Express 53 in binary notation. 1

Or

Write down the Cauchy's form of Remainder for Taylor's series in finite form.

Group – C (F.M. $65 = 25 \times 1 + 4 \times 10$)

Choose the correct answer

3. (i) The angle between the vectors $2\vec{\imath}+6\vec{\jmath}+3\vec{k}$ and $12\vec{\imath}-4\vec{\jmath}$ is

(a) 0^o (b) 45^o (c) 90^o (d) 180^o.

(ii) The triangle formed by the vectors $3\vec{\imath}-2\vec{\jmath}+\vec{k}$, $\vec{\imath}-3\vec{\jmath}+5\vec{k}$ and $2\vec{\imath}+\vec{\jmath}-4\vec{k}$ is a

(a) isosceles (b) equilateral (c) right angled (d) none of these.

(iii) If $\vec{a}.(\vec{b}\times\vec{c}) = 0$ then the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are

(a) coplanar (b) independent (c) collinear (d) none of these.

(iv) Find the area of the triangle whose vertices are $(1,-1,-3)$, $(3,-1,2)$ and $(4,-3,1)$

(a) 165 units (b) 82.5 units (c) 16.5 units (d) none of these.

(v) Every integer is one of form

(a) $2k-1, 2k+1, 2k-3, 2k+3$ (b) $3k, 3k-1, 3k+1, 3k-2, 3k+2$
(c) $4k+1, 4k+3, 4k+5$ (d) $5k, 5k-1, 5k+1$ (k is an integer).

(vi) The remainder when the sum $4! + 5! + 6! + \cdots + (50)!$ is divided by 4 is

(a) 1 (b) 2 (c) 3 (d) 0.

(vii) The remainder when 62 is divided by -8 is

(a) 2 (b) -2 (c) 0 (d) 6.

$(viii)$ The unit digit of 7^{40} is

(a) 1 (b) 9 (c) 3 (d) 7.

(ix) If n is a positive integer and n^3+1 is prime then n is equal to

(a) 1 (b) 2 (c) 11 (d) any odd integer.

(x) $a \equiv b (mod\ m)$ if and only if

(a) $a+b=pm$ (b) $a-b=pm$ (c) $ab=pm$ (d) $\frac{a}{b}=pm$, for some integer p.

(xi) $a \equiv b(mod\ m)$, then $a^n \equiv b^n(mod\ m)$ if and only if

(a) n is any integer (b) n is only positive integer (c) for $n<m$ (d) for integer $n>m$.

(xii) If $f(n) = 5^{2n+2} - 24n - 25$ then it is divisible by

(a) 476 (b) 576 (c) 5 (d) none of these.

$(xiii)$ The sequence $\{x_n\}$, where $x_n=(-1)^n$ oscillates finitely between

(a) -1 and 0 (b) -1 and 1 (c) 0 and 1 (d) 1 and 2.

(xiv) A sequence $\{x_n\}$ is said to be monotone increasing if

(a) $x_n \geq x_{n+1}$ (b) $x_n > x_{n+1}$ (c) $x_n \leq x_{n+1}$ (d) $x_n < x_{n+1}$.

(xv) The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if

(a) $p \geq 1$ (b) $p<1$ (c) $p>1$ (d) $p \leq 1$.

(xvi) The value of c in Rolle's theorem, where $-\frac{\pi}{2}<c<\frac{\pi}{2}$ and $f(x)=cosx$ is equal to

(a) 0 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) π.

$(xvii)$ The general solution of the differential equation $(D^3+3D^2+3D+1)y=0$ is

(a) $(a+bx+cx^2)e^{-x}$ (b) $(a+bx+cx^2)e^{x}$ (c) $(a+bx)e^{-x}$ (d) none of these.

$(xviii)$ $\frac{1}{D-1}x^2 =$

(a) x^2+2x+2 (b) $-(x^2+2x+2)$ (c) $2x-x^2$ (d) x^2-2x.

(xix) Charpit's auxiliary equation of the partial differential equation $z=pq$ are

(a) $\frac{dx}{p}=\frac{dy}{q}=\frac{dz}{-2pq}=\frac{dp}{-p}=\frac{dq}{-q}$ (b) $\frac{dx}{p}=\frac{dy}{-q}=\frac{dz}{2pq}=\frac{dp}{p}=\frac{dq}{q}$ (c) $\frac{dx}{-q}=\frac{dy}{-p}=\frac{dz}{-2pq}=\frac{dp}{-p}=\frac{dq}{-q}$
(d) none of these.

(xx) Equation $ptany+qtanx=sec^2z$ is of order

(a) 1 (b) 2 (c) 0 (d) none of these.

(xxi) The differential equation obtained by eliminating a and b from $z=ae^{pt}sinbx$ is

(a) $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial t}=0$ (b) $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial t^2}=0$ (c) $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial t^2}=0$ (d) none of these.

$(xxii)$ The criteria stated for Rolle's Theorem, Lagrange's Mean Value Theorem and Cauchy Mean Value Theorem are

(a) sufficient (b) necessary (c) both necessary and sufficient (d) sufficient but not necessary.

$(xxiii)$ The value of $\lim_{x\to 0}\frac{tanx-x}{x-sinx}$ is

(a) -1 (b) 0 (c) 1 (d) none of these.

$(xxiv)$ If c be a point in the interval in which the function $f(x)$ is defined and if $f'(c) = f''(c) = f'''(c) = \cdots = f^{n-1}(c) = 0$ and $f^n(c) \neq 0$, n be odd then

(a) $f(c)$ is a maximum (b) $f(c)$ is a minimum (c) $f(c)$ is neither a maximum nor a minimum (d) $f(c)$ is absolute maximum.

(xxv) The function $f(x, y)$ has maximum value at (a, b) if

(a) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0, AC > B^2$ and $A > 0$ (b) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0, AC > B^2$ and $A < 0$

(c) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0, AC < B^2$ and $A < 0$ (d) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0, AC = B^2$ and $A > 0$

$\left(A = \frac{\partial^2 f}{\partial x^2}, B = \frac{\partial^2 f}{\partial x \partial y}, C = \frac{\partial^2 f}{\partial y^2}\right)$.

Answer any four questions

4. (a) In a $\Delta\, ABC$, prove that $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$, where a, b, c and A, B, C have usual meanings. 5

(b) Prove by vector method that the line joining the middle points of two sides of a triangle is parallel to the third side and half of it. 5

5. (a) Using the principle of mathematical induction prove that $3^{2n+1} + 2^{n+2}$ is divisible by 7. 5

(b) Solve the Diophantine Equation $25x + 65y = 50$. 5

6. (a) Prove that there are infinite number of primes. 5

(b) Find the inverse of 19 modulo 141. 5

7. (a) Seven teams will participate in a Round –Robin tournament. Prepare a schedule of matches for this tournament. 5

(b) If $0—85312—612—x$ is the ISBN of a book, find the check digit. 5

8. (a) Test the series for convergence and divergence: $\frac{3}{2} + \frac{4}{2^2} + \frac{5}{2^3} + \cdots + \frac{n+2}{2^n} + \cdots$ 5

(b) Show that $\left\{\frac{5n+3}{4n+1}\right\}$ is a strictly decreasing sequence. 5

9. (a) Find the infinite series expansion of the function $f(x) = cosx$. 5

(b) Find a and b such that $\lim_{x\to 0} \frac{ae^x + be^{-x} + 2sinx}{sinx + xcosx} = 2$. 5

10. (a) Show that the maximum value of $x + \frac{1}{x}$ is less than its minimum value. 5

(b) Using Lagrange's method of undetermined multiplier find the stationary point of $V = x^2 + y^2 + z^2$, subject to the condition $x + y + z = 6$. Also determine whether V is maximum or minimum at this point, if possible. 5

11. (a) Solve by method of variation of parameters: $\frac{d^2y}{dx^2} + 4y = 4tan2x$. 5

(b) Find the complete integral of $z = px + qy + pq$. 5