BANKIM SARDAR COLLEGE

Part – II (1+1+1) Examination 2020

B.Sc. (General)

Subject: Mathematics (General)

Paper: II & III

Group: (1+2)

Time: 2 Hours | Full Marks: 25+25 = 50

(Answer each group in separate Answer-Sheets)

Group – 1

Paper – II (F.M.-25)

(Answer in separate Answer-Sheets)

Module – III & IV

Answer Question No. 1 and any two from the rest

1. State a necessary and sufficient condition for a non-empty subset of a group to be a subgroup of the group. Hence show that the set $\{2n: n \in \mathbb{Z}\}$ is a subgroup of the group $(\mathbb{Z}, +)$($\mathbb{Z}$ is the set of all integers). 5

Or

Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}$. 5

Or

Evaluate: $\lim_{x\to 0} \frac{x-sinx}{tan^2x}$. 5

2. Find the shortest distance between the lines $\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{2}$ and $5x - 2y - 3z + 6 = 0 = x - 3y + 2z - 3$. 10

3. If $u = logr$ and $r^2 = x^2 + y^2 + z^2$, prove that $r^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1$. 10

4. If $I_{m,n} = \int_0^{\frac{\pi}{2}} sin^m x cos^n x dx\,; m, n$ are positive integer > 1, then prove that $I_{m,n} = \frac{n-1}{m+n} I_{m,n-2}$. Hence find the value of $\int_0^{\frac{\pi}{2}} cos^4 x dx$. 10

5. Verify that the matrix $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{bmatrix}$ satisfies its characteristic equation. Also find the eigen values of A. 10

Group - 2

Paper – III (F.M.-25)

(Answer in separate Answer-Sheets)

Module – V & VI

Answer Question No. 6 and any two from the rest

6. Write down the following problem in standard form by introducing slack and surplus variables and identify them:

$$Max\ Z = 5x_1 + 8x_2$$
$$Subject\ to\ 3x_1 + 7x_2 \leq 18$$
$$-4x_1 + 6x_2 \geq 15;\ x_1, x_2 \geq 0.$$ 5

Or

A particle moves along a straight line according to the law $s = 6t^2 + 4t + 3$. Prove that the acceleration varies as $\frac{1}{s^3}$. 5

Or

Prove that $\Delta log f(x) = log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$. 5

7. A particle describes a plane curve under the action of a central attractive force F per unit mass. Prove that in usual notations the differential equation to the path of the particle is $\frac{h^2}{p^3}.\frac{dp}{dr} = F$. 10

8. Use Newton-Raphson method to find the real root of the equation $x^3 + 3x - 5 = 0$ correct to four significant figures. 10

9. Find the optimal assignment and the corresponding optimal cost of the following Assignment Problem: 10

	A	B	C	D
W	1	4	6	3
X	9	7	10	9
Y	4	5	11	7
Z	8	7	8	5

10. A particle describes a parabola $x^2 = 8y$ under a force always perpendicular to y-axis. Find the law of force and the velocity of the particle at any point of its orbit. 10