Bankim Sardar College

Group - A

Answer any one question

1. Prove that the matrix $A = \begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$ is orthogonal.   5

2. If $f(x) = x^2 - x$, then show that $f(y + 1) = f(-y)$.   5

3. Evaluate: $\lim\limits_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$.   5

Group – B

Answer any two questions

4. Solve by Cramer's Method: $x + y + z = 4; x - 2y + z = -2; 3x + 2y + 7z = 14.$ 10

5. If $x + y + z = 0$, show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0.$ 10

6. If $x^m y^n = (x + y)^{m+n}$, show that $\frac{dy}{dx} = \frac{y}{x}$. Hence show that $\frac{d^2y}{dx^2} = 0.$ 10

7. A firm produces x tones of output at a total cost of Rs. C, where $C = \frac{1}{10}x^3 - 5x^2 + 10x + 5$. At what level of output will the marginal cost and the average variable cost attain their respective minima?   10

8. Find by integration the area of the triangle bounded by the line $4y - 5x = 0$, the x-axis and the ordinate $x = 4$. Verify your result by using the definition of area of a triangle as half the product of the base and the altitude.   10

9. Evaluate: (i) $\int \frac{dx}{x\{(logx)^2 + 4logx - 12\}}$ (ii) $\int_0^1 \frac{(x+1)dx}{x^2+2x+3}.$ 5 + 5