

BANKIM SARDAR COLLEGE

Part - I (1+1+1) Examination 2020

B.Sc. (General)

Subject: Mathematics (General)

Paper: I

Time: 2 Hours

Full Marks: 25

Module - I & II

1. Show that the equation $\tan\left(i\log\frac{x-iy}{x+iy}\right) = 2$ represents the rectangular hyperbola $x^2 - y^2 = xy$. 5

Or

Solve by Cramer's rule: $x + y + z = 3$; $2x + 3y + 4z = 9$; $x + 2y - 4z = -1$.

2. Find the nature of the conic $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0$ by reducing it to canonical form. 5

Or

Under what condition that the straight line $r\cos(\theta + \alpha) = p$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$.

3. If $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are three vectors such that $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{0}$ and $|\vec{\alpha}| = 3, |\vec{\beta}| = 5, |\vec{\gamma}| = 6$, then show that $\vec{\alpha} \cdot \vec{\beta} + \vec{\beta} \cdot \vec{\gamma} + \vec{\gamma} \cdot \vec{\alpha} = -35$ 4

Or

A particle acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. Find the amount of work done.

4. If $y = \sin(m\sin^{-1}x)$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$. 5

Or

Show that the pedal equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with regard to the centre is $\frac{a^2b^2}{p^2} = a^2 + b^2 - r^2$.

Or

Use Raabe's Test to examine the convergence of the series $\sum u_n$, where $u_n = \frac{1.3.5 \dots (2n+1)}{2.4.6 \dots (2n+2)}$.

5. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$. 3

Or

Evaluate: $\lim_{n \rightarrow \infty} \left[\frac{1^2}{n^3+1^3} + \frac{2^2}{n^3+2^3} + \dots + \frac{n^2}{2n^3} \right]$

Or

Evaluate: $\int \frac{dx}{a \sin x + b \cos x}$

6. Solve: $\log\left(\frac{dy}{dx}\right) = ax + by$. 3

Or

Obtain the general and singular solution of $y = px + p^n$ where $p = \frac{dy}{dx}$.